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THE MODELLING OF SPIRAL-SEARCH TORPEDOES AND DEPTH BOMBS IN THE NAVAL WARGAMING SYSTEM

by

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The torpedo model in the Naval Wargaming System (NWGS) is described and modifications suggested to allow more accurate modelling of spiral-search torpedoes and depth bombs.

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THE MODELLING OF SPIRAL-SEARCH TORPEDOES AND DEPTH BOMBS IN THE NAVAL WARGAMING SYSTEM

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James N. Eagle

1. Introduction

The torpedo model in the Naval Wargaming System (NWGS) was described and evaluated in [1], and modifications were suggested to accommodate straight-running and snake-search torpedoes. In this technical report, additional changes are recommended to allow modelling of spiral-search torpedoes and depth bombs.

2. An Overview of the Existing Torpedo Model

The current NWGS torpedo attack model is summarized in the flow chart of Figure 1. Torpedo attacks may be launched at either a latitude/longitude point or (if the target is held on a sensor) at a track. In either case, an initial range check is made to determine if the target is within torpedo attack range. If not, an "out of range" message is passed to the attacker.

If firing at a point, a search is made in the vicinity of the point, and, if no target is found, the attacker is so informed. If firing at a track, a check is made to determine if the target is an aggregate. If so, the first target in the aggregate is selected.

Next a check is made to insure that the firing ship has sufficient torpedoes. If this check is successfully completed, the target is informed that he is under torpedo attack. A counter-attack is allowed if the target has torpedoes remaining and is in a "weapons free" status.

The program then calculates torpedo run time, run distance, and miss distance. If run time exceeds the maximum allowed run time for the torpedo being fired, a miss is awarded. Otherwise,

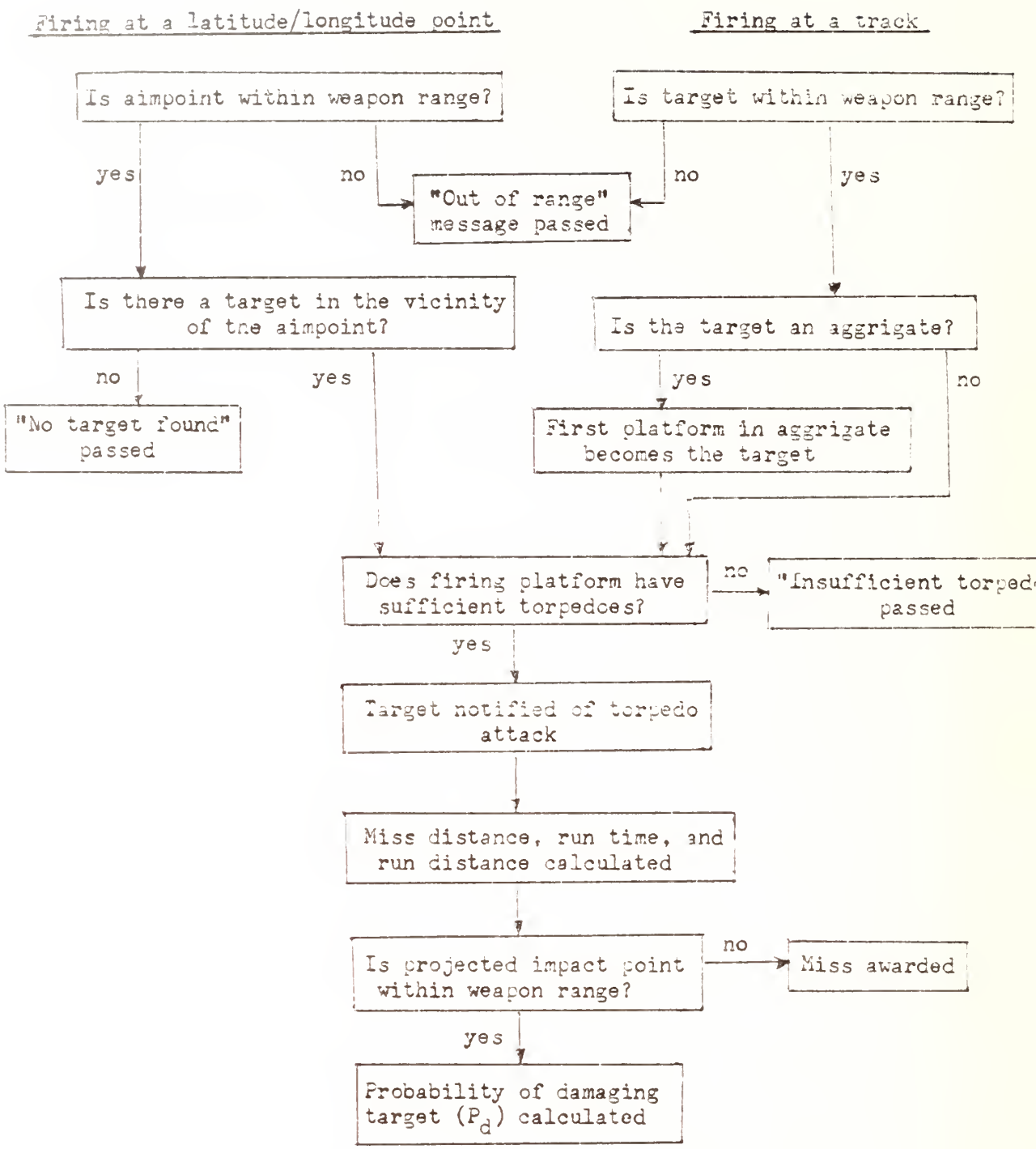


Figure 1. NWGS Torpedo Attack Model Flowchart.

the program continues and the probability of damaging the target (P_d) is calculated. For homing torpedoes, P_d is the product of the following terms:

1. angle factor,
2. sea-state factor,
3. weapon effectiveness factor,
4. detonation reliability factor,
5. cross-layer factors (2),
6. depth factor,
7. guidance reliability factor, and
8. target speed factor.

For straight-running torpedoes, only the first four terms are included in the product. A discussion of each factor is contained in [1].

3. Summary of Changes Recommended by [1]

Listed below are the major changes recommended by [1] to improve the existing model's representation of straight-running and snake-search torpedoes.

1. The degrading effect of the layer is eliminated except when a submarine target moves across layer depth after the torpedo is fired.

2. The target is not alerted of the torpedo attack if the target's speed is greater than some specified "maximum sonar speed".

3. The calculation of the total distance travelled by the torpedo (i.e., torpedo run) is modified to account for target motion while the torpedo is in the water.

4. The method of determining if an alerted and evading target out runs the torpedo is modified to more accurately model target motion.

5. A straight-running torpedo fired at a submarine submerged below periscope depth is assumed to miss the target. (The existing model allows a hit if the target changes depth by less than twice target height plus torpedo influence distance during torpedo run.)

6. For a homing torpedoes, P_d is the product of the following terms:

1. environmental (i.e., sea-state) factor
2. weapon reliability factor
3. cross-layer factor
4. target speed factor
5. weapon placement factor

The sea-state factor decreases, or at least does not increase with increasing sea-state. This models decreasing acoustic search performance with increasing sea-state.

The weapon reliability factor is the probability of all weapon hardware (including propulsion, guidance, and detonation systems) working properly. As [1] notes, this probability for various torpedoes is available in [2] and in fleet torpedo exercise reports.

The cross-layer factor is 1 unless the target is a submarine and changes depth across the layer during torpedo run. In this case, it decreases to .9.

The target speed factor reduces the probability of hit for slow targets. For echo ranging torpedoes, this models the difficulty of discriminating between low-doppler target echos and reverberations. For a passive torpedoes, it reflects the reduced radiated noise of slow targets and the corresponding reduction in torpedo search efficiency. The existing model sets

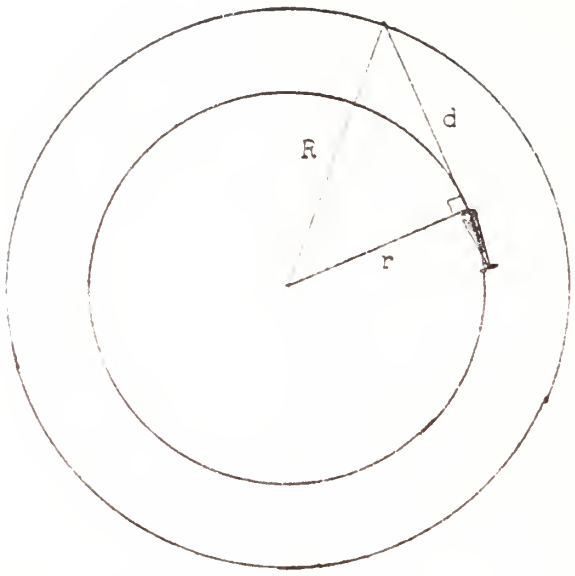
this factor to 1 when target speed is greater than 8 knots and to .8 for slower targets. This is not unreasonable, but a continuous increase for targets speeds increasing from 0 to 15 knots might be more realistic.

Finally, the weapon placement factor is the probability of accurate azimuthal placement of the torpedo. More specifically, it is the probability that a normally distributed angular firing error is less than or equal to half the apparent angular width of the target. For homing torpedoes, half the apparent angular width is the torpedo influence distance divided by target range. The effect of firing errors on P_d is not currently included in the NWGS torpedo model. The addition of the weapon placement factor does not overly complicate the calculations and captures more of the essence of the torpedo fire control problem.

4. Summary of Additional Changes Recommended to Model Spiral-Search Torpedoes and Depth Bombs

The existing NWGS torpedo model, as modified by [1], can be further changed relatively easily to accommodate spiral-search torpedoes and depth bombs. The recommended changes are as follows:

1. In the initial checks to determine if the target is within weapons range, the program M30-Range-Check-ee should set the maximum weapon range to the sum of maximum boost distance (if the weapon is rocket assisted) and maximum weapon influence radius. For a spiral-search torpedo, maximum weapon influence radius is $\sqrt{r^2 + d^2}$, where r is the radius of the spiral and d is



r = radius of spiral track
d = torpedo acoustic detection range
R = maximum weapon influence radius = $(r^2+d^2)^{\frac{1}{2}}$

Figure 2. Maximum Weapon Influence Radius for a Spiral Search Torpedo.

the torpedo's acoustic detection range. (See Figure 2.) For depth bombs, it is an increasing function of weapon yield.

2. When firing at a latitude/longitude point, the grid search procedure is modified to consider as potential targets all tracks which could come within one maximum weapon influence radius during the time the weapon is in the water.

3. The run time for depth bombs is set to 0.

4. The cross-layer factor is set to 1, since spiral-search torpedoes can be programmed to search in depth, and the damage effects of depth bombs are not strongly affected by acoustic layer depth.

5. The target speed factor is set to 1 for depth bombs, because acoustic homing is not used for these weapons.

6. The method in which the weapon placement factor (WPF) is calculated is modified to reflect the fact that these weapons are fired at points rather than along tracks.

The last change is probably the most fundamental, and the remainder of this report deals with it exclusively. In Section 5, background material on damage functions and diffuse Gaussian weapons is presented. In Section 6, these concepts are used to calculate the WPF and P_d for spiral-search torpedoes and depth bombs, and in Section 7 a numerical example is presented.

5. Damage Functions and Diffuse Gaussian Weapons

The damage function, D(r), for a weapon is the probability that the target is damaged to some specified level given the miss distance is r and the weapon operates (i.e., searches and

detonates) properly. The damage function is firing theory's analog to search theory's lateral range curve. Just as the area under the lateral range curve is called the "sweep width" of a sensor, the volume under $D(r)$ is the "lethal area" of a weapon. The damage function generally has no angular argument, θ, so that a radial symmetry of damage effects is implicitly assumed.

Perhaps the simplest weapon in concept is the "cookie cutter" weapon. The damage function for this weapon is 1 for all $r \leq R$ and 0 for all $r > R$. R is the "lethal radius" for this weapon, and the lethal area is πR^2.

Another popular weapon model, which is not as conceptually simple as the cookie cutter but which offers significant computational simplifications, is the diffuse Gaussian or Carleton weapon [3,4]. The damage function is

$$D(r) = \exp(-r^2/2b^2) ,$$

where b is a scale factor. Integrating over the (r,θ) plane gives a lethal area of $2\pi b^2$ for this weapon.

Figure 3 shows the damage function for cookie cutter and diffuse Gaussian weapons with the same lethal area. Between the two, the diffuse Gaussian weapon is probably the most appropriate model for depth bombs when there is significant uncertainty in either degree of target hardening, target aspect, or weapon yield. For modelling spiral-search torpedoes, the cookie cutter approximation may be slightly preferred; but for the level of modelling detail required by theater or battle group level games, either choice would probably be satisfactory.

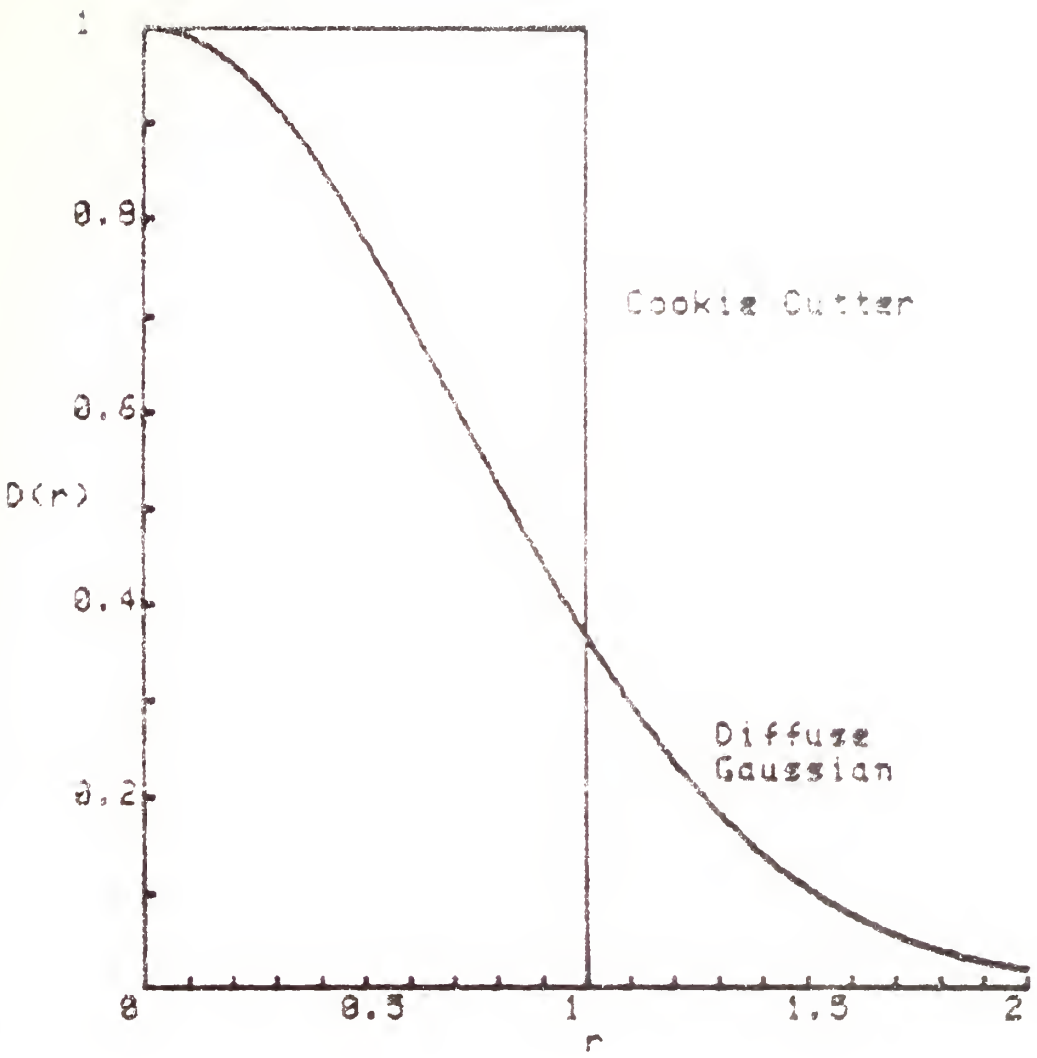


Figure 3. Damage Functions for Cookie Cutter and Diffuse Gaussian Weapons.

The advantage of the diffuse Gaussian weapon is not that it provides an especially accurate representation of damage effects. Rather, it is a reasonable approximation which, in addition, offers considerable computational simplifications when the target location errors and weapon firing errors have independent bivariate normal distributions. For example, consider the following problem:

1. A diffuse Gaussian weapon with lethal area $2\pi b^2$ is fired.

2. The weapon impact point (x,y), is given by a bivariate normal distribution with mean (μ_x,μ_y) and covariance matrix S_w .

3. Target location (u,v) is given by another bivariate normal distribution with mean (μ_u,μ_v) and covariance matrix S_t .

4. The only factor affecting the probability of damaging the target is the miss distance of the weapon.

In this case, the probability of damaging the target (P_d) reduces to what we call the weapon placement factor (WPF), which is found by integrating the product of the damage function and the probability distribution for miss distance over the plane. This integration yields

$$P_d = \frac{b^2}{[(b^2+\sigma_{E1}^2)(b^2+\sigma_{E2}^2)]^{1/2}} \exp\left[-1/2\left(\frac{(\mu_x-\mu_u)^2}{b^2+\sigma_{E1}^2} + \frac{(\mu_y-\mu_v)^2}{b^2+\sigma_{E2}^2}\right)\right], \text{ where}$$

$$\sigma_{E1}^2 = s_{22} \sin^2\theta - s_{12} \sin 2\theta + s_{11} \cos^2\theta ,$$

$$\sigma_{E2}^2 = s_{22} \cos^2\theta + s_{12} \sin 2\theta + s_{11} \sin^2\theta ,$$

$$\theta = 1/2 \tan^{-1}[2s_{12}/(s_{22} - s_{11})] , \text{ and}$$

$$\{s_{ij}\} = (S_t + S_w) .$$

The terms σ_{E1}^2 and σ_{E2}^2 are the variances of the miss distance distribution along the axes of the equiprobability ellipses, and θ is the angle between these axes and the coordinate system used to define S_w and S_t.

Evaluation of (1) can be considerably simplified in some common special cases. If, for example, the weapon is aimed at the center of the target distribution (which maximizes P_d and is thus the usual case when the target distribution is known by the firing platform), then $(\mu_x,\mu_y) = (\mu_u,\mu_v)$ and

$$P_d = \frac{b^2}{[(b^2 + \sigma_{E1}^2)(b^2 + \sigma_{E2}^2)]^{1/2}} .$$

If, in addition, both the target location and impact point distributions have errors in the line-of-sight (i.e., range errors) which are independent of errors across the line-of-sight (i.e., left-right errors), then S_t and S_w have 0 off-diagonal elements and θ is 0. In this case,

$\sigma_{E1}^2 = s_{11}$ = sum of variances of target location and impact point errors across the line-of-sight, and

$\sigma_{E2}^2 = s_{22}$ = sum of variances of target location and impact point errors in the line-of-sight.

Finally, if the target location and impact point distributions are circular normal with variances σ_t^2 and σ_w^2, and $(\mu_x, \mu_y) = (\mu_u, \mu_v)$, then

$$P_d = \frac{b^2}{b^2 + \sigma_E^2}$$

where $\sigma_E^2 = \sigma_t^2 + \sigma_w^2$.

In general, the components of the covariance matrices S_w and S_t should increase with increasing target range. This models the larger firing and localization errors associated with more distant targets.

Although (1) may not appear particularly simple, it is, in fact, a closed form solution for P_d. For an arbitrary damage function, D(r), and miss distance distribution, $f(r,\theta)$, the evaluation of P_d as

$$\int_0^\infty \int_0^{2\pi} D(r) f(r,\theta) \, d\theta \, dr$$

requires, in general, a numerical approximation.

6. Calculation of WPF and P_d

For the "point" weapons considered here, the weapons placement factor (WPF) is the probability that the horizontal distance between the target and the impact point is small enough to cause a specified level of damage. To be consistent with the general framework of the current NWGS torpedo model, there are four cases to consider:

1. Firing a single weapon at a latitude/longitude point.
2. Firing multiple weapons at a latitude/longitude point.
3. Firing a single weapon at a track.
4. Firing multiple weapons at a track.

Each case will be considered separately.

Firing a Single Weapon at a Latitude/Longitude Point

In this scenario, the player controlling the firing platform identifies an aim point and the program conducts a search in the vicinity of that point. If a target is found, a weapon may be fired at the specified point. Since the target is not identified with a track, it is presumably not held on a sensor and not being tracked. Thus the firing platform either does not have a target location distribution at which to direct its fire or has chosen to ignore available targeting information. To calculate WPF in this situation, (1) is evaluated with (μ_x, μ_y) set to the aim point, (μ_u, μ_v) the actual target position, and S_t a 2 x 2 matrix of zeros. P_d is then the product of the WPF and the other multiplicative terms discussed in Section 3.

Firing Multiple Weapons at a Latitude/Longitude Point

Assuming that all errors are independent (a reasonable assumption in this case), the multiple-weapon probability of damag

$$1 - (1 - P_d)^n ,$$

where P_d is the single-weapon probability of damage calculated above and n is the number of weapons fired in the salvo.

Firing a Single Weapon at a Track

This is probably the most common scenario. The firing platform is tracking the target with a sensor (or has received targeting information from another platform), and is assumed to have a target location distribution at which to fire. P_d is maximized when the firing platform aims at the center of this distribution. So WPF is calculated using (2).

In many instances, it is reasonable to assume that errors in the line-of-sight and across the line-of-sight are independent for both the target location and impact point distributions. (This is the case if and only if the equiprobability ellipses defining the two distributions are symmetric about the line-of-sight.) If so, the calculation of WPF is particularly simple, since θ is 0 and (3) is used to find σ_{E1}^2 and σ_{E2}^2. These calculations are illustrated by the example in Section 7.

Firing Multiple Weapons at a Track

If it is assumed that the firing platform does not relocalize the target prior to each individual shot in the salvo, then this

case is the most difficult one to analyze accurately. This occurs because the weapon miss distance for each shot in the salvo is the sum of an independent firing error plus a fixed but unknown location error. Thus to be the most accurately modelled, the miss distance can not be assumed to be selected from an independent distribution for each shot.

The most straightforward solution to this problem is simply to assume that the target is relocalized after each shot. Then the shots can be considered as probabilistically independent, and the single-weapon P_d can be used to calculate the multiple-weapon P_d as in (4). (This independence between shots is implicitly assumed in the current NWGS torpedo model.)

Alternatively, all the weapons in a single salvo can be considered as one large weapon with a lethal area equal to the sum of the individual lethal areas. Then (2) would be used to calculate WPF using the known distributions for weapon impact point and target location. P_d is then WPF times the other multiplicative factors of Section 3.

7. An Example

A ship launches a rocket-boosted spiral-search torpedo at a target being tracked at a range of 10 nautical miles (nm). At this range, the variance of target location errors are $.8nm^2$ across the line-of-sight and $.3nm^2$ in the line-of-sight. And weapon impact point errors have a variance of $.2nm^2$ across and $.4nm^2$ in the line-of-sight. So,

$$S_t = \begin{pmatrix} .8 & 0 \\ 0 & .3 \end{pmatrix},$$

$$S_w = \begin{pmatrix} .2 & 0 \\ 0 & .4 \end{pmatrix}, \text{ and}$$

$$S = S_t + S_w = \begin{pmatrix} 1 & 0 \\ 0 & .7 \end{pmatrix}.$$

The torpedo searches a spiral 1 nm in radius and has a 2 nm acoustic detection range. What is WPF if a diffuse Gaussian weapon is used to model the torpedo?

The lethal area of the torpedo is πR^2 where $R^2 = 1^2 + 2^2 = 5$. So $b^2 = 5\pi/2\tau = 2.5$, and

$$WPF = \frac{2.5}{[(2.5 + 1)(2.5 + .7)]^{1/2}}$$

$$= .747$$

The probability of damaging the target is then WPF times the environmental, weapon reliability, and target speed factors.

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